A-Soft Separation Axioms in Soft Topological Space

Luay Abd –Al-Hani Al-Sweedi **Zahraa Mohammad Najem Al-rekabi**
Department of mathematics ,College of Education for pure science
University of Babylon
Drluayha11@yahoo.com Zahraa90@yahoo.com

Abstract

The important science tools in a different kinds and specialties , that considered the basic mainstay of (the set theory) and because of huge development in all life fields. This causes great problems , that need solution and parallel tools for those developments , so the scientis become responsible to work on the development of number theory and open new horizons , that a new science had appeared which is
(soft figures theory) which is considered the important tool to solve most difficult problems or overcome them ,in these sciences and their specific life specialization, economy, medicine , geometry and others. Also the theory of soft numbers had entered in general topology in power full and active way. The last years a new science has appeared is (soft topological space).

The main idea of this research is to define the separation axioms in (soft topological space) and practically in certain point , and to study the most important properties and results of it .

Key words: Soft set, soft topology, $a-$ soft neighborhood, $a-$soft disjoint, $a-$soft continuous, $a-$soft separation axioms .

الخلاصة

ان اهم ادوات العلوم ومختلف اختصاصاتها وانواعها والتي تعتبر الركيزة الاساسية هي (نظرية المجموعــات) ونظــراً للتطور الهائل في جميع مجالات الحياة وهذا بدوره يولد مشاكل كبيرة ويحتاج الى حل والى ادوات موازية لتلك التطورات من اجل حلها، لذا أخذ العلماء على عاتقهم تطوي نظرية الاعداد وفتح افاق جديدة، حيث ظهر علم جديد هو (نظرة الأعداد الطرية) التــي أُعتبرت الاداة المهمة لحل معظم المشاكل المستعصية او تجاوزها في تلك العلوم ومختلف اختــصاصاتها الحياتيــة والاقتــصادية والهندسية والطبية وغيرها وكذلك دخلت نظرية الاعداد الطرية في علم التوبولوجيا العامة بصورة قوية وفعالــة فــي الــسنوات الاخيرة وظهر علم جديد هو (الفضاء التوبولوجي الطري).

ان الفكرة الاساسية من هذا البحث هي تعريف (بديهيات الفصل في الفضاء التوبولوجي الطري) وبوجه الخصوص عنــد نقطة معينة في المعلمات ودراسة اهم الخصائص والنتائج لها .

الكلمات المفتاحية: المجموعة الطرية ، التبولوجي الطري، الجوار الطرية-a ، الانفصال الطري- a، الاستمرارية الطرية-a، بديهيات الفصل الطريةa - .

Introduction

Life problems are many and various and need to be solved by human to make his life easier. Scientist didn't stop to find these solutions or trying to do that , in which scientist Molodtsov in 1999, defined (the soft set) which was regarded quality change and important mathematic tool , to solve these problems that it can't be solved using ordinary methods such as, fuzzy set, especially (mathematical function) membership. Also the problem that existed in most branches of Math., Geometry, economy and computer science and other sciences this field attracts many researchers to work deeply in studying features and laws of (soft sets theory) and developed it by, groups of researches

[Maji*et al.*,2003] the scientist begin to define a (topological space) based on (soft set theory) namely (soft topological space). Many groups of researchers and scientis have joined to study the new properties of this space and depending of the family of soft sets, every family depends on the concepts which were found (soft sets) or symbolizes (F_α) it is .

Through this definition, many types of families have appeared (i.e.g soft set). In [Cagman & Englinglu, 2011] the researcher depended on (soft set) to confirm function (F) and change its space which it is a subset of the set E of parameters. In[Yuksel *et al.*, 2014], The Turkish researcher made depend on the changing of function F and its domain is a subset of E , when he defined a soft set in [Xuechong, 2015], this Chinese researcher try to defined a new type of a soft sets called it (central soft sets) who present a new concepts that depend on set of (soft sets) basic map (F) on the a subset of (E), in [Molodtsov, 1999; Min, 2011 and Aygunoglu and Aygun , 2012, the researchers are defined the last kind of these sets which is going to the base of our study. This type depends on the change of a function F and confirming sets of parameters which is a subset of E. That's mean all functions on A to opposite $IP(X)$, that $A \subseteq E$ where the concept of (separation axioms) in soft topological spaces of last type were studies .

1. Soft sets

1.1. Definition [Molodtsov ,1999]

A pair (F,A) is called soft set over X , where F is a mapping defined as : $F: A \longrightarrow IP(X)$ such that $(F,A) = \{(a, F(a)), F(a) \in IP(X)\}$, where

1) X be the initial universal set , and E_X be the set of all possible parameters with respect to X .
2) $IP(X)$ is denoted to the power set of X (I . e $IP(X)$ the family of all subset of X .
3) $A \subseteq IP(X)$, s-imply we denoted for E_X by E .
4) The set of all soft sets over the universe X is denoted by $S(X)$.

1.2. Definition [Molodtsov ,1999]

If $F(a) = X$ for all $a \in A$ then , the soft set F_A is called the absolute soft set and it is denoted by $\tilde{X}_A$. If $F(a) = \varphi$ for all $a \in A$ then the soft set F_A is called the null soft set . and it is denoted by $\tilde{\Phi}_A$.

1.3. Note

let F_A is any soft set over the universe X , then :

1. The point of the soft set F_A at $a \in A$ is denoted by F_a such that $F_a = \{(a, F(a)\}$
2. for $\alpha \in A$ and $\forall x \in X$, a soft point, x_a is of the form: $x_a = \{(a,x)\} \cup \{(p,\varphi); \forall p \in A, p \neq a\}$, simply we write the soft point by $x_a = \{(a,x)\}$.
3. A soft point x_a belong to the soft set F_A and denoted by $x_a \tilde{\in} F_A$ or $x_a \tilde{\subseteq} F_A$ if and only if $x \in F(a)$ and $x_a \tilde{\notin} F_A$ if and only if $x \notin F(a)$.
4. An $a -$ absolute soft set is of the form $\tilde{X}_a = \{(a,\{X\})\}$, and is called the element of the absolute soft set $\tilde{X}_A$ at $a \in A$.

5. An a −null soft set is of the form $\tilde{\Phi}_{\mathrm{a}} = \{(\mathrm{a},\{\varphi\})\}$, and is called the element of the null soft set $\tilde{\Phi}_A$ at $a \in A$.
6. An e −soft complement is an e −soft set over χ defined as : $\tilde{X}_A -_a F_A = H_a$ *such that* $H_a = \{(a, X - F(a))\}$.
7. We denoted to the soft set F_A at the point $a \in A$ by F_a .

1.4. Note [Georgion, 2013]

Let $F_A \tilde{\in} S(X)$, $a \in A$ and $x \in X$, then we write $x \in_a F_A$ and $x \notin_a F_A$ resp. if and only if $x \in F(a)$ and $x \notin F(a)$.

1.5. Note

If F_A , G_A be are soft sets over the universe X, then:

(1) F_A is an $\mathrm{a}-$ soft subset of G_A if and only if $F(a) \subseteq G(a)$ and we write by the form $F_A \tilde{\subseteq}_a G_A$. for $a \in A$.

(2) The a-soft intersection of F_A and G_A , is an a −soft set defined as follows : $F_A \tilde{\cap}_a G_A = H_a$ *such that* $H_a = \{(a, F(a) \cap G(a))\}$.

(3) The a −soft union of F_A *and* G_A , is an a −soft set defined as follows : $F_A \tilde{\cup}_a G_A = H_a$ such that $H_a = \{(a, F(a) \cup G(a))\}$.

(4) F_A and G_A are called a −soft equal iff $F(a) = G(a)$, and denoted by $F_A =_a G_A$.

(5) The a −soft disjoint of F_A *and* G_A is defined by $F_A \tilde{\cap}_a G_A = \tilde{\Phi}_a$.

(6) A point x is a −belong to the soft set F_A is denoted by $x \tilde{\in}_a F_A$, that is $x \in F(a)$.

Let $\wedge$ be any arbitrary index set , and let F_{λ_A} be any soft set over X , we define :

(1) $\cap_{a\, \lambda \in \wedge} F_{\lambda_A} = H_a = \{(a, \cap_{a\, \lambda \in \wedge} F_\lambda(a))\}$

(2) $\cup_{a\, \lambda \in \wedge} F_{\lambda_A} = H_a = \{(a, \cup_{a\, \lambda \in \wedge} F_\lambda(a))\}$

1.6. Proposition

F_A , G_A be any two soft sets over the universe X, for each $a \in A$ the following statements are hold :

(1) $F_A \tilde{\cap}_a \tilde{\Phi}_A = \tilde{\Phi}_a$

(2) $F_A \tilde{\cap}_a \tilde{X}_A = F_a$

(3) $F_A \tilde{\cup}_a \tilde{\Phi}_A = F_a$

(4) $F_A \tilde{\cup}_a \tilde{X}_A = \tilde{X}_a$

We can conclude these facts directly from [Note 1.5] it can be considered as a special case of [proposition 2.3] in [P. k . Maji , R. Bis was and A .R . Roy (2003)] .

1.7. Proposition

If F_A , G_A and H_A are any soft sets over the universe X , for each $a \in A$ the following are true :

1) $F_A \tilde{\cap}_a (G_A \tilde{\cap}_a H_A) = (F_A \tilde{\cap}_a G_A) \tilde{\cap}_a H_A$
2) $F_A \tilde{\cup}_a (G_A \tilde{\cup}_a H_A) = (F_A \tilde{\cup}_a G_A) \tilde{\cap}_a H_A$
3) $F_A \tilde{\cap}_a (G_A \tilde{\cup}_a H_A) = (F_A \tilde{\cap}_a G_A) \tilde{\cup}_a (F_A \tilde{\cap}_a H_A)$
4) $F_A \tilde{\cup}_a (G_A \tilde{\cap}_a H_A) = (F_A \tilde{\cup}_a G_A) \tilde{\cap}_a (F_A \tilde{\cup}_a H_A)$.

Remark1.8

All of theorems and properties that are true in [Cagman *et al.*, 2001; Cagman and Englinglu ,2011; Cagman and Englinglu ,2011] are also be true in this research (i.e when we considered the point $a \in A$ as a base of this work) .

1.9. Definition

Let X and Y be two universal sets and A ,B be a sets of parameters , u:X→Yand p:A⟶B , then the mapping :

f: $(\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\rho}, B)$ (i .e $f: S(X) \longrightarrow S(Y)$) on A and B respectively is denoted by f_{pu} and it can shows as :

$$f_{pu} = \{(f_{pu}(F_A), p(A), p(A) \subseteq B)\}$$

$$\text{Where : } f_{pu}(F_A)(\beta) = \begin{cases} u\left\{ \bigcup_{\alpha \in p^{-1}(\beta)} (\mathcal{F}(\alpha)) \right\}, & \text{if } P^{-1}(\mathcal{B}) = \varphi \\ \varphi & \text{other wise} \end{cases}$$

For $\mathcal{B} \in B\ \exists\ a \in p(A)$ such that $p(a) = \mathcal{B}$. that is $p^{-1}(\mathcal{B}) \neq \varphi$
$A\ \ p^{-1}(\beta) \subseteq A$, hence $p^{-1}(\beta) \cap A \neq p^{-1}(\beta)$, hence we get that

$$f_{pu}(F_A)(\beta) = u\left\{ \bigcup_{\alpha \in p^{-1}(\beta)} F(\alpha) \right\}$$

Constructing :

Since p *is* a mapping , *so* $p(A) \neq \varphi$, $\forall A \neq \varphi$, that is $\forall \mathcal{B} \in p(A) \exists\ a \in A$ such that $P(a) = \mathcal{B}$ *and* $P^{-1}(\mathcal{B}) \neq \varphi$ now , $a \in P^{-1}(P(a))$ *so ,*

$$f_{pu}(F_A)(\mathcal{B}) = u\left\{ \bigcup_{\alpha \in p^{-1}(\beta)} (F(\alpha)) \right\} \forall \beta \in P(a)\} \ .$$

- If p is a one to one , then $P^{-1}(P(A))$ =A , *that is* $\forall \mathcal{B} \in P(A)\ \exists\ a \in A$ such that $P(a) = \mathcal{B}$ *and* $f_{pu}(F_A)(\mathcal{B}) = u(F(a))$
- If G_B $S(Y)$ then the inverse image of G_B under f_{pu} is denoted by $f_{pu}^{-1}(G_B)$ is a soft set $F_A \in S(X)$ such that:
  $P(a) = u^{-1}(G(P(a)))$ for every $a \in A$.

1.10. Remark

For each $a \in A$ and $x \in X$, then we can defined the soft mapping f_{pu} at a soft point x_a , is :

$$f_{pu}(x_a)(\mathcal{B}) = u(\bigcup_{\alpha \in p^{-1}(\mathcal{B})} x_a(\alpha)) = u(x)$$

$$I .e \ \left(f_{pu}(x_a)\right)(p(a)) = u(x)$$

$I.e\ \left(f_{pu}(x_a)\right)_{p(a)} =$ {(P(a), {u(x)})}

Now , for $b \in B$ and $y \in Y$, $f_{pu}^{-1}(y_b)(a) = u^{-1}(y)$ for $b = p(a)$.

proposition 1.11 [Zorlutuna *et al.*, 2012]

" let $F_{1_A}, F_{2_A} \in S(X)$ and $G_{1_B}, G_{2_B} \in S(Y)$, the following statements are true

i) $if\ F_{1_A} \tilde{\subseteq} F_{2_A}$, then $f_{pu}(F_A) \tilde{\subseteq} f_{pu}(F_{1_A})$.

ii) $G_{1_B}, \tilde{\subseteq} G_{2_B}$, then $f_{pu}^{-1}(G_{1_B}) \tilde{\subseteq} f_{pu}^{-1}(G_{2_B})$

iii) $F_{1_A} \tilde{\subseteq} f_{pu}^{-1}\left(f_{pu}(F_{1_A})\right)$

iv) if p is a one to one map of X into Y and u is a one to one map of A into B, then $F_{1_A} = f_{pu}^{-1}\left(f_{pu}(F_{1_A})\right)$.

v) $f_{pu}\left(f_{pu}^{-1}(G_{1_B})\right) \tilde{\subseteq} G_{1_B}$.

Remark 1.12

For any $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ and $a \in A, b \in B$, the flowing statements are true :

(1) $x_{1_a} \tilde{\subseteq} x_{2_a}, f_{pu}(x_{1_a}) \tilde{\subseteq} f_{pu}(x_{2_a})$

(2) $y_{1_b} \tilde{\subseteq} y_{2_b}, f_{pu}^{-1}(y_{1_b}) \tilde{\subseteq} f_{pu}^{-1}(y_{2_b})$

(3) $x_{1_a} \tilde{\subseteq} f_{pu}^{-1}(x_{1_a})$

(4) $f_{pu}\left(f_{pu}^{-1}(y_{1_b})\right) \tilde{\subseteq} y_{1_b}$

(5) If p *is* a map from X into Y, and u is a map of A into B,

then: $f_{pu}\left(f_{pu}^{-1}(y_{1_b})\right) = y_{1_b}$.

(6) $f_{pu}^{-1}(\tilde{X}_b - y_{1_b}) = \tilde{X}_a - f_{pu}^{-1}(y_{1_b})$

2. Soft topology

In this section we defines some of important concepts of a soft topological spaces with some of counter examples .

2.1. Definition

Let X be an initial universal set , and $A \subseteq E$ be a set of parameters ,

Let $\tilde{\tau}$ be a subfamily of a the family of all soft sets $S(X)$, *we say that* the family $\tilde{\tau}$ is a soft topology on X if the following axioms are holds :

(1) $\tilde{\Phi}_A, \tilde{X}_A \tilde{\in} \tilde{\tau}$

(2) if $F_A, G_A \tilde{\in} \tilde{\tau}$, then then $F_A \tilde{\cap} G_A \tilde{\in} \tilde{\tau}$

(3) For any index set I $G_{i_A} \tilde{\in} \tilde{\tau}$, for any $i \in I$, then $\tilde{\cup}\{G_{i_A}, i \in I\} \tilde{\in} \tilde{\tau}$.

The triple $(\tilde{X}_A, \tilde{\tau}, A)$ is called soft topological space or (soft space) .

*The m*embers *of* $\tilde{\tau}$, *are* called *s*oft *o*pen *s*ets , A *s*oft set F_A is called *s*oft *c*losed set *i*f *a*nd *o*nly *i*f *i*ts *c*omplement *i*s *s*oft *o*pen , *T*he family *o*f all *s*oft *c*losed set *i*s *d*enoted *b*y :

$$C(\tilde{X}_A) = \{\tilde{X}_A - F_A, F_A \tilde{\in} \tilde{\tau}\}$$

2.2. Definition [Georgion 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space ,a∈A , $x \in X$, we say that a soft set $F_A \tilde{\in} \tilde{\tau}$ is an a−soft open nhd of x in $(\tilde{X}_A, \tilde{\tau}, A)$ if $x \in F(a)$, simply we denoted by $G_{(a,x)}$.

Simply we write the (neighborhood) by nhd .The set of all soft (nhd) of a point $x \in X$ at $a \in A$ is denoted by $N_{\tilde{\tau}(a,x)}$, that is $N_{\tilde{\tau}(a,x)} = \{G_{(a,x)}; G_{(a,x)}$ is soft nhd of a soft point $x_a\}$.

2.3. Definition

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space $a \in A$, we say that the soft set FAis a-soft nhd of x ∈X if there exists a soft open set $G_{(a,x)}$ such that $x \in_a G_{(a,x)} \tilde{\subseteq}_a F_A$.

A soft set F_A is called $a-$soft conhd of $x \in X$ if and only if $\tilde{X}_A - F_A$ is $a-$soft nhd of $x \in X$.

A soft set F_A is called $a-$soft locally open set if and only if F_A is $a-$soft nhd of each $x \in_a F_A$.

A soft set F_A is called a−soft locally closed set if and only if $\tilde{X}_A - F_A$ is $a-$soft locally open .

Proposition2.4

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , The a−soft nhd system of a point x has the following properties :

(1) if $G_{(a,x)} \in N_{\tilde{\tau}(a,x)}$ then $x \in_a G_{(a,x)}$.

(2) if $G_{(a,x)} \in N_{\tilde{\tau}(a,x)}$ and $G_{(a,x)} \tilde{\subseteq}_a H_A$,then $H_A \tilde{\in} N_{\tilde{\tau}(a,x)}$.

(3) if $G_{(a,x)}$ and $H_{(a,x)} \in N_{\tilde{\tau}(a,x)}$, then $G_{(a,x)} \tilde{\cap} H_{(a,x)} \in N_{\tilde{\tau}(a,x)}$.

The proof of them is directly from definition [2.2]

3. Soft separation axioms

3.1. Definition

Let $(\tilde{X}_A, \hat{\tau}, A)$ be a soft topological space , let $a \in A$, the soft space is called a−soft $T_\circ -$ space if for each $x_1 \neq x_2$ in X , there is a a−soft open set $G_{(a,x_1)}$ containing x_1 but not containing x_2 or there is a soft open set $G_{(a,x_2)}$ containing x_2 but not containing x_1 .

The soft topological space is called soft $T_\circ -$ space if $\forall a \in A$, the soft space is a −soft $T_\circ -$ space..

3.2. Note

If $(\tilde{X}_A, \tilde{\tau}, A)$ is a−soft $T_\circ -$ space for some point $a \in A$, then the soft space need not be a soft $T_\circ -$ space .

Example 3.3

Let $A \subseteq E$ such that $A = \{a_1, a_2\}$ and let X be a universal set such that : $X = \{x_1, x_2, x_3\}$, note that :

$\hat{\tau} = \{F_{1_A}, F_{2_A}, F_{3_A}, F_{4_A}, F_{5_A}, F_{6_A}, \tilde{\Phi}_A, \tilde{X}_A\}$ *where :*

$F_{1_A} = \{(a_1, \{x_2\}), (a_2, \{\varphi\})\}$, $F_{2_A} = \{(a_1, \{x_1, x_2\}), (a_2, \{x_3\})\}$,

$F_{3_A} = \{(a_1, \{x_1, x_3\}), (a_2, \{x_1, x_2, x_3\})\}$, $F_{4_A} = \{(a_1, \{x_2\}), (a_2, \{x_1, x_2\}\}$

$F_{5_A} = \{(a_1, \{x_1\}), (a_2, \{x_3\})\}$, $F_{6_A} = \{(a_1, \{x_1, x_3\}), (a_2, \{x_1, x_2\})\}$

then $(\tilde{X}_A, \tilde{\tau}, A)$ is $a_1 -$ soft $T_\circ -$ space .

Now :$\{(a_2, \{x_1\})\} \neq \{(a_2, \{x_2\})\}$ and $\nexists$ a_2 –soft open set such that its contain one and not contain the other .

Thus $(\tilde{X}_A, \tilde{\tau}, A)$ is not soft $T_\circ -$ space .

3.4. Definition [Georgion, 2013]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , for $a \in A$, the soft space is called a –soft $T_1 -$ space if for each $x_1 \neq x_2$ in X , there is two soft open sets $G_{(a,x_1)}$ and $H_{(a,x_2)}$ such that : $x_1 \in_a G_{(a,x_1)}$ and $x_2 \notin_a G_{(e,x_1)}$ so $x_2 \in_a H_{(a,x_2)}$ and $x_1 \notin_a H_{(a,x_2)}$.

The soft topological space is called soft $T_1 -$ space if and only if $\forall a \in A$ the soft space $(\tilde{X}_A, \tilde{\tau}, A)$ is an $a -$ soft $T_1 -$ space , [D.N .Georgion 2013] .

3.5. Note

$a -$ soft $T_1 -$ space need not be soft $T_1 -$ space .

3.6. Example

the example (3.3) elucidate that the soft space is $a_1 -$ soft $T_1 -$ space but not $a_2 -$ soft $T_1 -$ space , so it is not soft $T_1 -$ space ..

3.7. Definition [Georgion, 2013]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , and $a \in A$, the soft space is called a –soft $T_2 -$ space if for each $x_1 \neq x_2$ in X , there is two $a -$ disjoint soft open sets $G_{(a,x_1)}$, $H_{(a,x_2)}$.

The soft space $(\tilde{X}_A, \tilde{\tau}, A)$ is called strongly soft $T_2 -$ space if it is a –soft $T_2 -$ space if for each $a \in A$.

3.8. Note

a –soft $T_2 -$ space need not be a soft $T_2 -$ space .

3.9. Example

The example (3.3) elucidate that the soft topology is a_1 –soft $T_2 -$ space , but it is not a_2 –soft $T_2 -$ space , so it is not soft $T_2 -$ space .

3.10. Definition [Georgion, 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively , $x \in X$ and p:A⟶B .A map u:X⟶Y is called soft p−continuous at the point x if every a ∈ A and p(a)−soft open nhd G_B of u(x) there exist an a−soft open nhd F_A of x such that $f_{pu}(F_A) \tilde{\subseteq} G_B$.

3.11. Proposition [Georgion, 2014]

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively ,and p: $A \rightarrow B$ and u: $X \longrightarrow Y$, then u is soft p−continuous iff $f_{pu}{}^{-1}(G_B) \tilde{\in} \tilde{\tau}$, for every $G_B \tilde{\in} \tilde{\rho}$.

3.12. Definition

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively ,then for a ∈ A and x ∈ X, the map $f_{pu}(\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\rho}, A)$ is called an a −soft continuous at x iff for each p(a) −soft open set G_B containingg u(x) there exists an a −soft open set H_A containing x, such that $f_{pu}(H_A) \tilde{\subseteq}_{p(a)} G_B$.

f_{pu} is called soft continuous if it is a −soft continuous $\forall a \in A$.

3.13.Definition [Zorlutuna, 2012]

A soft mapping f_{pu} from a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ into a soft topological space $(\tilde{Y}_B, \tilde{\rho}, A)$ is a soft continuous iff $f_{pu}{}^{-1}(G_A) \tilde{\in} \tilde{\tau}$ for any $G_A \tilde{\in} \tilde{\rho}$.

3.14. Definition [Nazmul, 2012]

Let $(X_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively , the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\rho}, A)$ is called soft open map if for each soft open set G_A in $\tilde{X}_A$,then $f_{pu}(G_A)$ is soft open set in $\tilde{Y}_B$.

3.15. Definition

Let $(X_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\rho}, A)$ are two soft topological spaces over X and Y respectively , the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\rho}, A)$ is :

(a) One to one if p and u are ono to one maps .

(b) Onto if p and u are onto maps .

3.16. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and $\tilde{X}_A$ be $a - \text{soft} - T_\circ - \text{space}$ for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is soft open map and u, p are onto maps , then $\tilde{Y}_B$ is a $p(a) - \text{soft} - T_\circ - \text{space}$.

Proof //

Let $b \in B$ and $y_1 \neq y_2$ in Y then there exist $e \in A$ and $x_1 \neq x_2$ in X such that $p(e) = b$,$u(x_1) = y_1$ and ,$u(x_2) = y_2$ because u, p are onto maps .Now , Since $(\tilde{X}_A, \tilde{\tau}, A)$ is $a-\text{soft}-T_1-\text{space}$, then there exist two soft open sets G_{1_A} and G_{2_A} such that : $x_1 \in_a G_{1_A}$ and $x_2 \notin_a G_{1_A}$ or $x_2 \in_a G_{2_A}$ $x_1 \notin_a G_{2_A}$ and , then

$y_1 = u(x_1) \in_b f_{pu}(G_{1_A})$

$y_2 = u(x_2) \notin_b f_{pu}(G_{1_A})$. or

$y_2 = u(x_2) \in_b f_{pu}(G_{2_A})$ and , $y_1 = u(x_1) \notin_b f_{pu}(G_{2_A})$,

So $(\tilde{Y}_B, \tilde{\sigma}, B)$ is $b-\text{soft}-T_\circ-\text{space}$.

Theorem 3.17

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and u:X$\rightarrow$Y is a one to one soft is $p-$ continuous map , if $\tilde{Y}_B$ is a $b-\text{soft}-T_\circ-\text{space}$ for some $b \in B$ and p is onto mapping , then there exist $a \in A$ such that $b = p(a)$ and $\tilde{X}_A$ is a$-\text{soft}-T_\circ-\text{space}$."

"Proof//

Since p is onto map and $b \in B$, so $\exists a \in A$ such that $b = p(a)$, now let $x_1 \neq x_2$ in X, since u is a one to one map , then $u(x_1) \neq u(x_2)$ in Y, but $\tilde{Y}_B$ is a b$-\text{soft}-T_\circ-\text{space}$, so there exist a soft open set $G_{(b,u_b(x_1))}$ containing $u_b(x_1)$ but not contain $u_b(x_2)$, hence $f_{pu}^{-1}(u_b(x_1)) \tilde{\in}_a f_{pu}^{-1}\left(G_{(b,u_b(x_1))}\right)$ and $f_{pu}^{-1}(u_b(x_2)) \tilde{\notin}_a f_{pu}^{-1}\left(G_{(b,u_b(x_1))}\right)$, but $f_{pu}^{-1}(u_b(x_1)) = x_{1_a}$ and $f_{pu}^{-1}(u_b(x_2)) = x_{2_a}$ because u is a one to one map , also f_{pu} is p-soft continuous , so by [definition 3.11] $f_{pu}^{-1}\left(G_{(b,u_b(x_1))}\right)$ is soft open set containing x_{1_a} but not x_{2_a} therefore $(\tilde{X}_A, \tilde{\tau}, A)$ is a$-\text{soft}-T_\circ-\text{space}$.

Theorem 3.18

Every soft subspace of a$-\text{soft}-T_\circ-\text{space}$ is a$-\text{soft}-T_\circ-\text{space}$ $\forall a \in A$.

Proof//

Suppose that $\tilde{Y}_A$ is a soft subspace of the of the $a-\text{soft}-T_\circ-\text{space}$ $(\tilde{X}_A, \tilde{\tau}, A)$, and $x_1, x_2 \in_a \tilde{Y}_A$ with $x_1 \neq x_2$.

Since $\tilde{Y}_A \tilde{\subseteq} \tilde{X}_A$, then $x_1, x_2 \in_a \tilde{X}_A$, but $\tilde{X}_A$ is a$-\text{soft}-T_\circ-$space , then there exist a soft open set $G_{(a,x_1)}$ such that $x_1 \in_a G_{(a,x_1)}$ and

$x_2 \notin_a G_{(a,x_1)}$ or there exist a soft open set $H_{(a,x_2)}$ such that $x_2 \in_a H_{(a,x_2)}$ and $x_1 \notin_a H_{(a,x_2)}$, since $x_1 \in_a G_{(a,x_1)}$ and $x_1 \in_a \tilde{Y}_A$, then

$x_1 \in_a G_{(a,x_1)} \tilde{\cap} \tilde{Y}_A$ and since $x_2 \notin_a G_{(a,x_1)}$ then $x_2 \notin_a G_{(a,x_1)} \tilde{\cap} \tilde{Y}_A$ or

$x_2 \in_a H_{(a,x_2)}$ and $x_2 \in_a \tilde{Y}_A$, then $x_2 \in_a H_{(a,x_2)} \tilde{\cap} \tilde{Y}_A$, and since $x_1 \notin_a H_{(a,x_2)}$, then $x_1 \notin_a H_{(a,x_2)} \tilde{\cap} \tilde{Y}_A$ so , $\tilde{Y}_A$ is a$-\text{soft}-T_\circ-$space .

3.19. Definition

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces , we say that $\tilde{\tau}$ is finer than $\tilde{\sigma}$ if and only if $\tilde{\sigma} \subseteq \tilde{\tau}$.

3.20. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces such that $\tilde{\sigma}$ is finer than $\tilde{\tau}$, if $\tilde{\tau}$ is $a-\text{soft}-T_\circ-\text{space}$, then $\tilde{\sigma}$ is $e-\text{soft}-T_\circ-\text{space}$ for some $a \in A$.

Proof // the prove is directly .

3.21. Theorem

A soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is a –soft-$T_1-\text{space}$, $a \in A$ iff there is a sodt point x_a is a –soft locally closed set for each $x \in X$.

Proof //

Let $x \in X$ we must prove that x_a is a–soft locally closed set , now let y $\in_a \tilde{X}_A - x_a$, y $\in X - \{x\}$, thus $y \neq x$, then there exists two soft open set $G_{(a,y)}$ and $H_{(a,x)}$ such that y $\in_a G_{(a,y)}$ and $x \notin_a G_{(a,y)}$ and $x \in_a H_{(a,x)}$ and $y \notin_a H_{(a,x)}$, hence $y \in G(a) \subseteq X - \{x\}$, implies that $G_{(a,y)} \tilde{\subseteq} \tilde{X}_A - x_a$, hence $\tilde{X}_A - x_a$ is a –soft nhd of each $y \in_a \tilde{X}_A - x_a$, that is $\tilde{X}_A - x_a$ is a –soft locally open set , hence x_a is a –soft locally open set .

Conversely : suppose that for each $x \in X$ x_a is a –soft locally open set , we want to prove that $(\tilde{X}_A, \tilde{\tau}, A)$ is a –soft-$T_1-\text{space}$, let $x, y \in X$, such that $x \neq y$, that is x_a and y_a are –soft locally closed sets , this implies that $\tilde{X}_A - x_a$ and $\tilde{X}_A - y_a$ are a –soft locally open sets and evidently that $x \in_a \tilde{X}_A - y_a$ and $y \notin_a \tilde{X}_A - y_a$ and $y \in_a \tilde{X}_A - x_a$ and $x \notin_a \tilde{X}_A - x_a$ so there exists a soft open sets G_A and H_A such that $x \in_a G_A \tilde{\subseteq}_a \tilde{X}_A - y_a$ and $y \in_a H_A \tilde{\subseteq}_a \tilde{X}_A - x_a$ that is $x \notin_a G_A$ and $x \notin_a H_A$, hence $\tilde{X}_A$ is a –soft-$T_1-\text{space}$.

3.22. Theorem

Let $(\tilde{Y}_B, \tilde{\sigma}, B)$ is b –soft-$T_1-\text{space}$ for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space , such that the maps $u: X \longrightarrow Y$ be onto and $p: A \longrightarrow B$ is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is a –soft-$T_1-\text{space}$ if $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is a –soft continuous.

3.23. Theorem

A soft topological space$(\tilde{X}_A, \tilde{\tau}, A)$ is *an* a –soft-$T_1-\text{space}$ if and only the soft point x_a is a –soft locally closed set for each $x \in X$.

3.24. Theorem

A soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$ is a–soft– $T_\circ$–space for some $a \in A$ if and only if ∀ x≠y in X we have $Cl(x_a) \neq Cl(y_a)$.

3.25. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and $\tilde{X}_A$ be $a-\text{soft}-T_1-\text{space}$ for some $a \in A$, if the map $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is e –soft open map and u, p are onto maps , then $\tilde{Y}_B$ is a $p(a)-\text{soft}-T_1-\text{space}$.

3.26. Theorem

Every soft subspace of a – soft T_1 – space is a– soft T_1 – space .

Proof // the proof is directly from Definition 3.4

27. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft $topological$ space , then the following statements are equivalent :

a) $(\tilde{X}_A, \tilde{\tau}, A)$ is $a-\mathrm{soft}-\mathrm{T}_1-\mathrm{space}$.

b) an $a-$ soft intersection of all the $a-$soft open nhd of an arbitrary soft point of x_a is singleton point $\{x\}$.

Proof//

The prove is directly by Note 1.5 and Definition 3.4 .

Note 3.28

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_A, \tilde{\sigma}, A)$ be two soft topological spaces such that $\tilde{\sigma}$ is finer than $\tilde{\tau}$, for some point $a \in A$ if $(\tilde{X}_A, \tilde{\tau}, A)$ is $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$, then $(\tilde{Y}_B, \tilde{\sigma}, B)$ is $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$.

3.29. Theorem

Every soft subspace of $a-$soft $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$ is $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$

Proof // the prove is directly by Definition 3.7 .

3.30. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft topological space , for a $\in$A the following statements are equivalent :

a) $(\tilde{X}_A, \tilde{\tau}, A)$ is $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$.

b) The a$-$ soft intersection of all $a-$soft closed nhds of an arbitrary $a-$soft point of x_a is $a-$ singleton point .

proof// The proof is directly from definition Note 1.5 and Definition 3.4

3.31. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces and $\tilde{X}_A$ be $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$ for some $e \in A$, if the maps $f_{pu}: (\tilde{X}_A, \tilde{\tau}, A) \longrightarrow (\tilde{Y}_B, \tilde{\sigma}, B)$ is soft open and u ,p are onto maps , then $\tilde{Y}_B$ is $p(a)-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$.

Proof // simply combine .

3.32. Theorem

Let ($(\tilde{Y}_B, \tilde{\sigma}, B)$ be a $b-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$ for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space such that the map u be onto and p is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is $a-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$, if f_{pu} is a $-$soft conyinuous .

3.33. Theorem

Let ($(\tilde{Y}_B, \tilde{\sigma}, B)$ be a $b-\mathrm{soft}-\mathrm{T}_\circ-\mathrm{space}$ for $b \in B$ and let $(\tilde{X}_A, \tilde{\tau}, A)$ be any soft topological space such that the map $u: X \longrightarrow Y$ be onto and $p: A \longrightarrow B$ is a one to one , then there exists $a \in A$ with $p(a) = b$ and $\tilde{X}_A$ is $a-\mathrm{soft}-\mathrm{T}_\circ-\mathrm{space}$, if f_{pu} is a $-$soft conyinuous .

3.34. Theorem

Let $(\tilde{X}_A, \tilde{\tau}, A)$ and $(\tilde{Y}_B, \tilde{\sigma}, B)$ be two soft topological spaces such that the maps u is one to one soft p $-$continuous and p is onto map .If for some $b \in B$, $\tilde{Y}_B$ is

$\mathrm{b}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$, then there exists $a \in A$ such that $b = p(a)$ and $\tilde{X}_A$ is $\mathrm{a}-\mathrm{soft}-\mathrm{T}_2-\mathrm{space}$.

References

Aygunoglu, A., and Aygun, H., 2012, Some Notes on Soft Topological Spaces , Journal Computer and applied, 21 ,113-119 .

Ahmad, B. and Athar Kharal,2009, On Fuzzy Soft Sets, Hindawi publishing corporation in fuzzy system , Article ID 586507, 6 pages .

Molodtsov, D.A., 1999, soft set theory- first results ,computers and mathematics with applications, 37 ,19-31.

Georgiou, D.N., Megaritis, A.C., and Pertropoulos, V.I.,2013, on soft topological spaces , Applied math. And inf . science 7 , No .5 , 1889-1901.

Georgiou, D.N. and Megaritis , A.C., 2014, soft set theory and topology , App. Gen. top.1, 93-109 .

Xuechong, G., 2015, Study on central soft sets :definitions and basic operations (2015) .

Zorlutuna, I., Akdag, M., Min, W.K., and Asmaka, S., 2012, Remarks on soft topological spaces , annals of fuzzy mathematics and informations , 3,171-185 .

Ali, M. I., *et al*., 2009, on some new operations in soft set theory , computers and mathematics with applications , 75 ,1547-1553 .

Shabir, M., and Naz, M., 2011, on soft topological spaces , computers and mathematics with applications , 611780-1799 .

Cagman, N., and Englinglu , S .S., 2011, soft topology , computers and mathematics with applications , 62 ,351-358 .

Maji, P.K., Biswas, R., and Roy, A.R.,2003 , soft set theory , computers and mathematics with applications , 45, 555-562.

Nazmuland, S.K., Samanta, S.K., 2012, Neighborhood of soft topological spaces , Ann.fuzzy Math . inf. 1-16 .

Yuksel , S., Tuzlu, N., and Ergul , Z. G.,2014 , soft regular generalized closed sets in soft topological spaces , Int. journal of math .Analysis, 8 (355-367) .

Min, W.K., 2011,a note on soft topological spaces , computers and mathematics with applications , 62 ,3524-3028 .